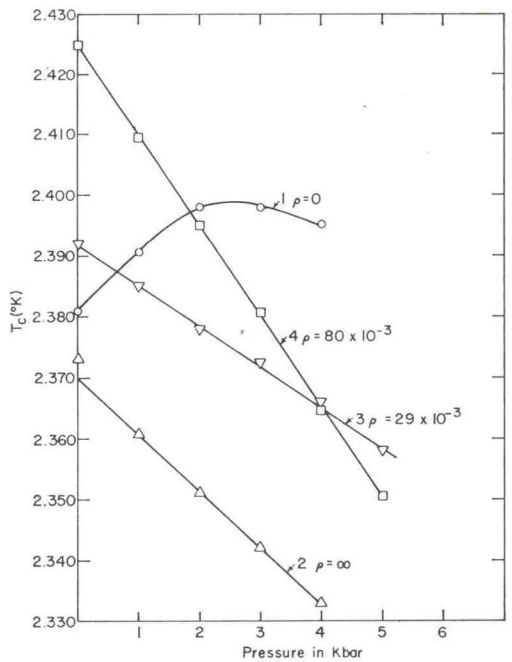

FIG. 7. Analysis of the pressure-induced enhancement of T_c of thallium in terms of pressure-dependent gap anisotropy. Description in the text.

tells us that the best choice is $\chi/\rho\lambda=1900$, which differs by only about 10% from our calculated value and thus confirms it.

We are now ready to present numerical data for the anisotropy parameter $\lambda\langle a^2\rangle$. From the straight-line fits of Fig. 4 and with $\chi/\rho\lambda=1700$, we obtain the curve shown in Fig. 6, which is our main result. In this plot, data on a second sample are included. The interesting physical feature is the monotonic rise of the anisotropy of the energy gap from a rather small value of $\lambda\langle a^2\rangle=0.0008$ at zero pressure up to $\lambda\langle a^2\rangle=0.0069$ at 4 kbar, with an indication of a saturation at higher pressures. This result is the basis for our explanation of the enhancement of the transition temperature in thallium in the lower pressure region. We wish to point out that since $\lambda\langle a^2\rangle$ is inversely proportional to $\chi/(\rho\lambda)$, changes in the choice of $\chi/(\rho\lambda)$ affect only the absolute values of $\lambda\langle a^2\rangle$ for the various pressures, but not the fact that the gap anisotropy increases markedly with pressure.

Before discussing the results we wish to draw attention also to the detailed agreement over a wide range of ρ between our data at 4 kbar, where they provide the most sensitive test, and the exact function $I_c(\chi)$ of the theory of Markowitz and Kadanoff. Allowance for the fact that thallium is a stronger coupling superconductor has been made in calculating $I_c(\chi)$. With $\chi/(\rho\lambda)=1700$, $\lambda\langle a^2\rangle=0.0069$, and $\partial T_c/\partial\rho=0.72$, we have good agreement of the function $I_c'(\rho)$, shown as a heavy solid line in Fig. 4, and the experimental data, except for one point at the upper end where the high resistivity ratio $\rho=80\times10^{-3}$ was obtained in a different way.

The fact that our experimental data seem to follow the theory in such detail leads us to believe that the effects of the generation of lattice imperfections by cold-working due to the application of high pressure are appropriately represented as effects on the gap anisotropy of shortening the electronic mean free path. It also shows that the valence effect is linear for $\rho<30\times10^{-3}$.

Although it seems already evident from Fig. 2, we now try to analyze more quantitatively how the anomalous enhancement of T_c under pressures up to 2 kbar can be explained by the observed dependence of the anisotropy of the energy gap in thallium. In Fig. 7, curve 1 represents the pressure dependence of T_c for the limit $\rho\to0$ as obtained from the extrapolation in Fig. 3. From this curve we subtract the maximal depression of T_c due to complete smoothing out of the gap anisotropy by going to the limit $\rho\to\infty$, which is given by Eq. (44c) of Ref. 9:

$$(\Delta T_c)_{\max}=T_c\frac{1}{VN(0)}\langle a^2\rangle.$$

We then arrive at curve 2 of Fig. 7, which shows the linear decrease common for soft superconductors. This curve shows what we would have for the anisotropy effect alone as $\rho\to\infty$. We obtain an average slope

$$(dT_c/dp)_{\rho\to\infty}=-1.0\times10^{-5}\ ^\circ\text{K/kbar}.$$

For a quantitative check of our anisotropy concept we would like to have the slope dT_c/dp for $\rho\to\infty$ measured. This condition cannot be verified experimentally, and in fact in this limit the theory also should fail.[9] We therefore compare our calculated slope $(dT_c/dp)_{\rho\to\infty}$ with the slope for the two highest resistivity ratios obtained, $\rho=29\times10^{-3}$ and 80×10^{-3}. The data for these are included in the figure as curves 3 and 4. We see that the calculated $\rho\to\infty$ slope is already approached by the slope of the lower experimental ρ, but the higher experimental-ρ slope differs by a factor of 1.5. It must be borne in mind, however, that experimentally the highest resistivity ratio was obtained by forcing the sample through a phase transformation which certainly affected at least the valence effect, as already indicated in Fig. 4, so that this datum probably should be disregarded.[16]

We therefore conclude that the concept of pressure-dependent gap anisotropy discussed above quantitatively explains the anomalous enhancement of the transition temperature of thallium at pressures up to 2 kbar.

An interesting question remains of how this large pressure dependence of the gap anisotropy can be

[16] In fact, there is indication from the data that the valence effect becomes pressure-dependent at this high imperfection concentration. This dependence cannot be isolated, however, if it is due to the different scattering centers introduced by the different procedure or if at higher concentrations $\partial T_c/\partial\rho$ becomes intrinsically pressure-dependent.

explained. Gap anisotropy as treated above contains, by definition, anisotropy in the phonon spectrum, in the electron-phonon interaction, and in the electronic density of states. Which of these factors is affected most by pressure cannot be decided without further investigation. It may very well be that the anisotropy in the density of states dominates. Associated variation of the energy dependence of the density of states was assumed by Lazarev and co-workers[4–7] to explain their data in a qualitative way.

COMPARISON WITH OTHER RESULTS

It is easy to see that our results are not in contradiction with the data of Quinn and Budnick,[17] who did not observe a decrease of T_c in thallium upon decreasing the electronic mean free path by dilute alloying of indium, bismuth, and lead, but observed an immediate increase of T_c. In Fig. 8 we have plotted the dependence of the transition temperature on the residual resistivity ratio as obtained in our experiments on plastically deformed pure thallium at zero pressure, together with Quinn and Budnick's results on dilute alloys of In, Pb, and Bi. In spite of the paucity of the experimental points for the alloys, it seems obvious that the large valence effect in the alloys obscures the very small anisotropy effect in thallium. In our results on plastically deformed thallium, the valence effect is just strong enough to eliminate any decrease in T_c as ρ decreases, leaving only a weakly curved region at small ρ. At zero pressure, the gap anisotropy is known to be small from an investigation of Weil and Lawson, who determined the value of the energy gap for several directions by ultrasonic methods.[18] From their data, we roughly estimate $\langle a^2 \rangle$ to be around 0.0007 compared with our value of 0.0008.

Hasse and Lüders[19] and Hasse and Seifritz[20] have obtained varying resistivity ratios and accompanying changes in T_c by deforming Tl plastically by twisting wires at cryogenic temperatures. There is strong disagreement between our results and theirs, though they used methods which at first sight seem closely related to ours. The discrepancies are most easily demonstrated by replotting their data together with ours, Fig. 9. Curves 1–6 represent the twisted-wire data on various samples. Since they were obtained at zero pressure, these data are to be compared with our zero-pressure curve (open circles), which was identical on two samples. Besides the large differences between different samples, the curvature and the enormous decrease in T_c are most striking. Although we do not think that substantial residual stresses in the twisted wires can be held responsible for these discrepancies, we include our data obtained under a pressure of 4 kbar for comparison. Without further information, we can only give a guess about the source of the differences: Since Hasse and Seifritz find good agreement of their twisted-wire data with alloy data on indium,[20] the suspicion arises that thallium is anomalous for some reason. It is conceivable that there is an influence of a phase transformation.[8]

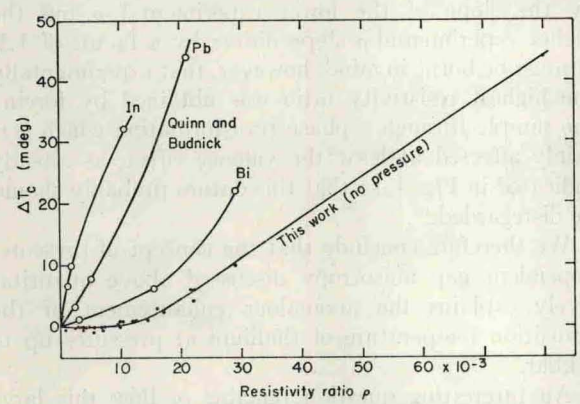

FIG. 8. Comparison of our data, obtained by plastic deformation on two samples at zero pressure, with data of Quinn and Budnick (Ref. 8) obtained by dilute alloying of In, Pb, and Bi.

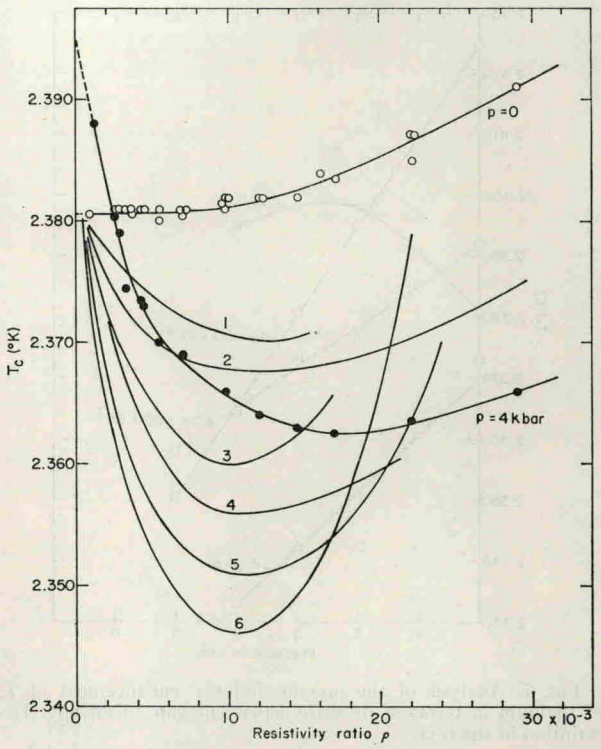

FIG. 9. Comparison of the data obtained by different methods of plastic deformation on the change of T_c in thallium with resistivity ratio. Curves 1–6: different samples, twisted-wire technique (Refs. 19, 20). Open circles: data on two samples, present work, no pressure, lattice imperfections introduced by repeated application of high pressure. Full circles: the same, measuring pressure 4 kbar.

SUMMARY

We conclude that the picture of suppression of gap anisotropy, as the electron mean free path is decreased,

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[18] R. Weil and A. W. Lawson, Phys. Rev. **141**, 452 (1966).
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